BASIC CONCEPTS OF DISPLACEMENT OR STIFFNESS METHOD:

INTRODUCTION

As discussed previously in this method nodal displacements are the basic unknowns.

Equilibrium equations in terms of unknown nodal displacements and known stiffness coefficients (force due a unit displacement) are written.

These equations are solved for nodal displacements and when the nodal displacements are known the forces in the members of the structure can be calculated from force displacement relationship.

DISPLACEMENT

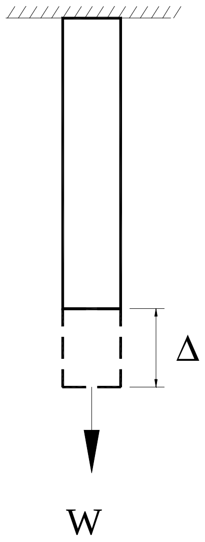
OR

STIFFNESS METHOD

2.2 STIFFNESS, STIFFNESS COEFFICIENT AND STIFFNESS MATRIX:

The stiffness of a member is defined as the force which is to be applied at some point to produce a unit displacement when all other displacement are restrained to be zero.

If a member which behaves elastically is subjected to varying axial tensile load (W) as shown in fig. 2.1 and a graph is drawn of load (W) versus displacement (Δ) the result will be a straight line as shown in fig. 2.2, the slope of this line is called stiffness.



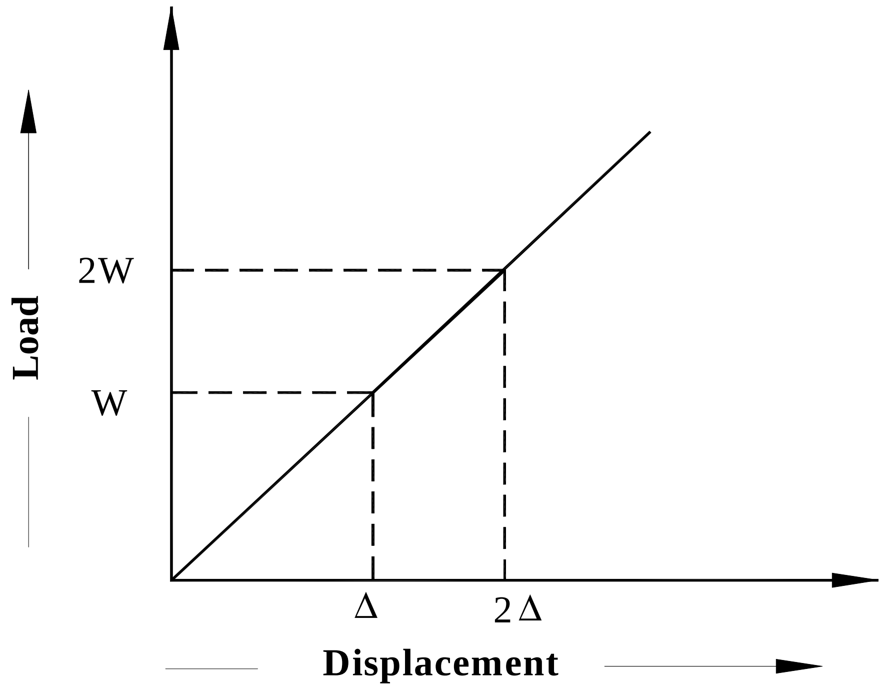


FIG.2.1

(Members subjected to varying axial load)

FIG.2.2

(Graph of load verses displacement)

Mathematically it can be expressed as

$$K = W/\Delta \qquad \text{------ } 2.1$$

In other words Stiffness 'K' is the force or load required at a certain point to cause a unit displacement at that point.

Equation 2.1 can be written in the following form

$$W = K\,\Delta \qquad \text{----- } 2.2$$

Where,

W = Force at a particular point

K = Stiffness

Δ = Unit displacement of the particular point.

The above equation relates the force and displacement at a single point. This can be extended for the development of a relationship between load and displacement for more than one point on a structure.

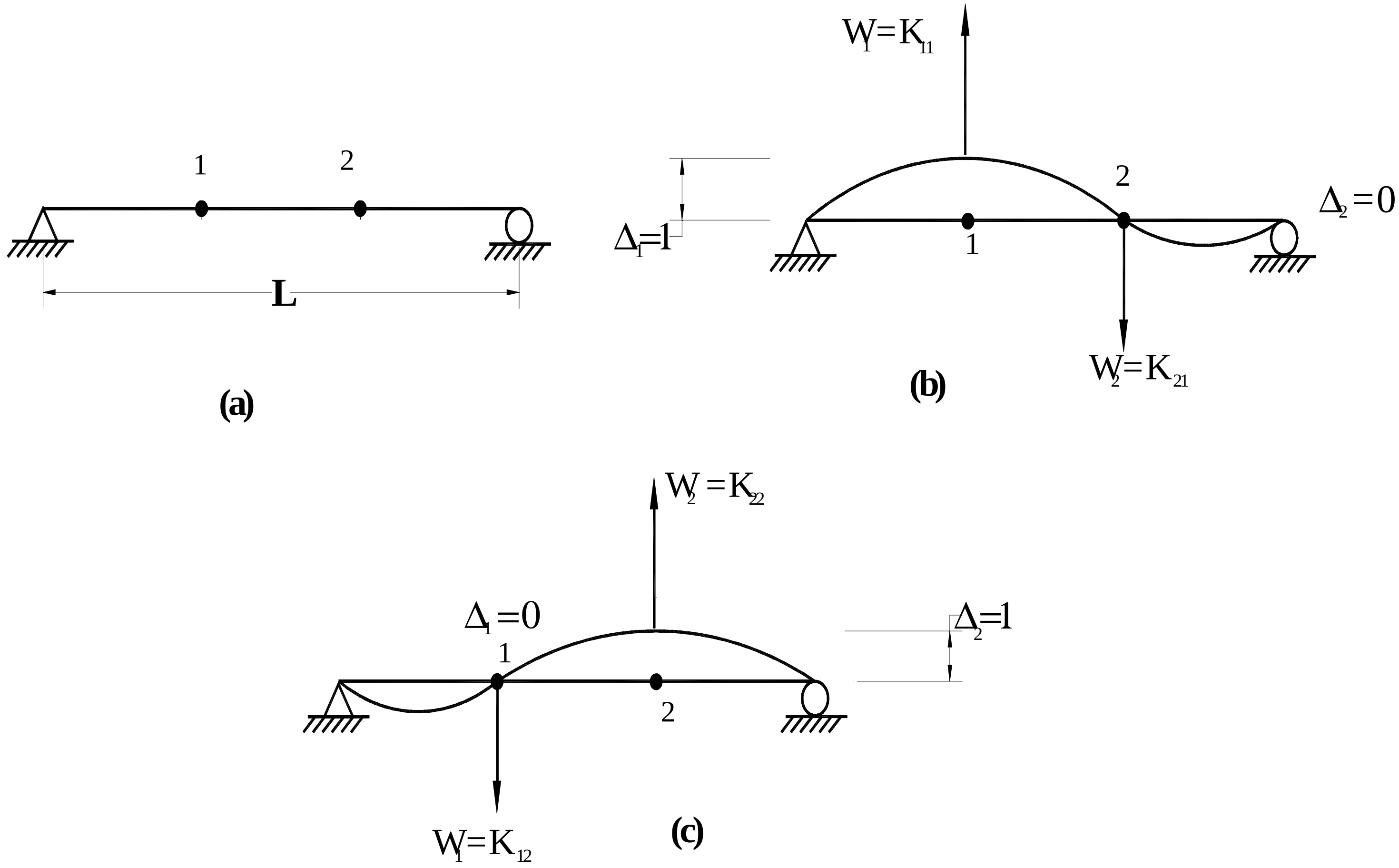


Fig:2.3

Let us consider a beam of fig. 2.3 and two points (nodes) 1, and 2. If a unit displacement is induced at point '1' while point '2' is restrained from deflecting up or down (see the definition of stiffness). then the forces "W1"and "W2" can be expressed in terms of "Δ1" in equation 2.2 as:

$$W = K\,\Delta \qquad \text{----------} \ (2.2)$$

when Δ1 = 1

$$W1 = K11.\,\Delta 1 = K11$$

See fig. 2.3(b)

$$W2 = K21.\,\Delta 1 = K22$$

where,

K11 = force at 1 due to unit displacement at 1

K21 = force at 2 due to unit displacement at 1

These are known as stiffness co-efficients.

If a unit displacement is induced at a point "2" while point "1" is restrained from deflecting up or down,then the forces W1 and W2 can be expressed in terms of " Δ2" in equation 2.2 as;

when Δ2 = 1

W1 = K12. Δ2 = K12

See fig. 2.3(c)

W2 = K22. Δ2 = K22

where,

K12 = force at 1 due to unit displacement at 2

K22 = force at 2 due to unit displacement at 2

First subscript indicates the point of force and second the point of deformation.

The following equation for the beam of fig.2.3 can be written as

W1 = K11 Δ1 + K12 Δ2 ---------- (2.3)

W2 = K21 Δ1 + K22 Δ2 ---------- (2.4)

Rewriting this in matrix form

$$\begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} \quad \text{--------- (2.5)}$$

where

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$ **is called *stiffness* matrix.**

Elements of the stiffness matrix are known as stiffness Coefficients and $\begin{bmatrix} W_1 \\ W_2 \end{bmatrix}$ ***is called force vector***

and $\begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix}$ **is called *displacement or deformation vector*.**

The expression (2.5) expresses the equilibrium at each of the node points in terms of stiffness co-efficients and the unknown nodal deformation and can be written as:

$$W = K\,\Delta \qquad \text{----------} \quad (2.6)$$

The matrix K contains the stiffness co-efficients and it relates the forces W to the deformations Δ and is called stiffness matrix. W and Δ are called force and deformation vectors.